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Taylor Based Jump Diffusion Model of Fractional Brownian Motion of Stock Price

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Abstract: Assuming that the stock price follows the stochastic differential equation driven by fractional Brownian motion and jump process, under the condition that the interest rate and volatility are constant. It has nothing to do with the relative importance of the information causing the jump. Generally speaking, the relative jump height of stock price depends on the relative importance of the important information. Fractional Brownian motion not only has various properties of fractional Brownian motion, but also has nonstationary increment. On the basis of fractional Brownian motion model, the fractional Brownian motion model with jump, namely fractional jump-diffusion process, is investigated. It is proposed to use Taylor expansion to deal with the control items in the model. For large-scale numerical simulation, it can reduce the amount of calculation required by the algorithm and save computer memory. The method of guessing value function and Taylor series expansion is introduced to solve the value function. The dialectical relationship between mathematics and finance is shown from one side: on the one hand, mathematics is a powerful tool for financial research; on the other hand, financial practice has promoted the development of mathematical theory itself.
Keywords: Stock price, fractional Brownian motion, jump diffusion process.

Introduction

The price of an option is essentially a risk price. There are many factors that affect the price of an option, including the current price of the underlying asset, the execution price of the option, the volatility of the underlying asset and the risk-free interest rate [1]. The pricing model has been demonstrated under different assumptions, and it is found that this model is not particularly consistent with the actual situation when simulating the stock market, because the stock price is not only related to the current price but also to the past price [2]. Adding more complex jump variables to the model makes it more consistent with the time length and jump height of stock market emergencies affecting stock prices, making the model more suitable for large-scale data processing. The distribution of asset prices is not a standard lognormal distribution, and the distribution of stock prices has the characteristics of sharp peaks and fat tails, which means that geometric Brownian motion cannot fully reflect the movement process of stock prices [3]. To effectively manage risks, we must correctly price financial derivative securities. How to determine the fair price of financial derivative securities is the key to their reasonable existence and healthy development? The principle of fair premiums, without any market assumptions, proves that when the stock price obeys the Brownian motion, the actuarial pricing and the no-arbitrage pricing are consistent [4].

Capital market is a fractal market, a stable, self-similar structure created by global certainty and local randomness, and the randomness of fractal time series in local [5]. In the real capital market, when some important information appears, the stock price will jump discontinuously, so the jump-diffusion process can better describe the financial market [6]. When the interest rate in the whole economy increases, the expected growth rate of stock price also tends to increase. However, the current position of future cash flow received by option holders will decrease. This affects the value of put options. Therefore, with the increase of risk-free interest rate [7]. The relative jump height of stock price is related to the relative importance of the important information causing the jump, and generally the size of the relative jump height depends on the relative importance of the important information [8]. The present value and the strike price of the stock's ending value when discounted at the expected rate of return Expectation [9]. The use of fractional Brownian motion instead of geometric Brownian motion makes up for the shortcomings of using geometric Brownian motion to describe the process of stock price. In order to improve the numerical simulation effect and simplify the algorithm, we use Taylor expansion to approximate it as a low-order polynomial. Therefore, this article analyzes Taylor's stock price fraction Brownian motion jump-diffusion process model [10].

Option pricing of stock price in jump diffusion process

The time length and jump height of the stock market emergencies on the stock price make the model more suitable for the processing of large-scale data. At the same time, for the convenience of numerical simulation, it is an extremely effective processing method for the parameters and discrete variables. There are often abnormal changes out of proportion to the continuous changes, showing an intermittent "jump" process. The continuous changes can be regarded as normal changes brought about by some normal conditions in the economy. The seller of an option will bear a certain potential risk during the term of the option. If he wants to add insurance to this risk, his premium is the price of the option, that is to say, the price of the option will be measured by the amount of risk the other party bears. If we use fractional Brownian motion to describe the financial market, there will be great deviation. Fractional Brownian motion has its own unique properties, that is, non-stationary increment, and it also has all the properties of fractional Brownian motion. The results are all based on other fundamental changes that remain unchanged. In particular, when the interest rate rises (falls), the stock price also falls (rises). If the net effect of the interest rate change and the subsequent stock price change is considered, the opposite may be obtained.

Modification of fractional jump diffusion process model

If the jump form in the fractional jump diffusion process model is introduced into the generalized compensated Poisson process, the time ductility of the jump form can be well reflected. The average growth rate of stock price caused by Poisson jump. The process of stock movement is divided into two parts: one is the “normal” price fluctuation, that is, the arrival of some small information makes the stock price fluctuate a little. It is assumed that the stock price is a stochastic differential equation driven by fractional Brownian motion and jump diffusion process, and the volatility is constant. The value of call options and European put options does not necessarily increase. This is because the execution opportunities of long-term options do not necessarily include all the execution opportunities of short-term options. Long-term options can only be executed on the maturity date. On this basis, it is modified to a process that can control the jumping frequency, and the expected return rate and volatility of the stock are further improved to be continuous non-random functions of time and stock price. The overall statistical structure, which is random and has long-term correlation, has its own trend and cycle: uneven digestion of information, non-linear reflection of information leads to biased random walk.

The change process of stock price only obeys the random process of normal state, which can’t reflect the whole process of stock price change, but the introduction of Poisson jump process can better reflect this jump mutation. Integrable function, the number of random jumps, it is an independent parametric process; The height of each stock jump is a random variable. From the conditions in the definition, we can easily see that the random events are counted from time zero, of which the counting process is the most direct cause, i.e. the total number of random occurrences of time during the period of increment. There are two different forms of changes in stock prices. In most of the time, the changes in stock prices are local changes, i.e. if the stock prices are in the state of 0 at the moment and are changing to the state of 0+1 or 1, this change is equivalent to the diffusion of continuous time situations. Holders of American options that are usually in the real position are most irritated by holding the option until its expiry date, rather than executing it immediately. This shows that the option also has a time value, and the full value of the option is the sum of the intrinsic price and the time value. If the stochastic process is both an independent incremental process and a stable incremental process, then we call it an independent stable incremental process as shown in Figure 1.

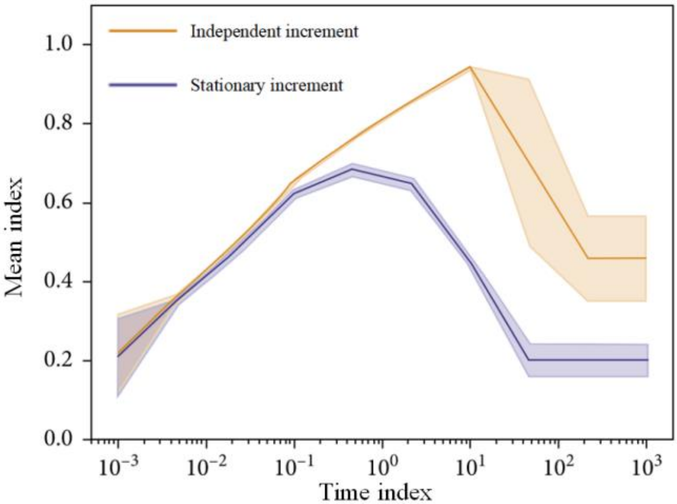

Figure 1. Independent stationary incremental process

Exchange option pricing in fractional Brownian motion

Financial market mathematics in the context of fractional Brownian motion After the exercise of equity warrants, the company will issue new shares, there will be new shareholders, and the company's shares will increase, which will have an impact on the company's capital structure, earnings per share, and stock price. If the jump form in the fractional jump diffusion process model is introduced into the generalized compensated Poisson process, the time ductility of the jump form can be well reflected. Because of the difficulty in solving the stochastic partial differential equation, the form of guess value function and Taylor series expansion are introduced to solve the value function. Finally, the analytic expression of the approximate solution of the optimal configuration is obtained. From an economic point of view, this shows the neutrality of risks. In fact, it shows that in the probability of equivalence. The expected return rate of risky assets at any given time is the same as that of risk-free securities. In order to control the jumping intensity and prevent the factors that are too large and too small and not suitable for the stock market situation, a scaled jumping variable is introduced into the model.

Adding more complex jump variables to the model are mutually independent and identically distributed random variables, indicating that at random time, the height at which a jump occurs at any time specifies the state when no jump occurs, is independent and obeys the variance of normal distribution, instead of unconditional expectation. If random variables in a random process always have the same probability distribution, then another kind of risk asset, called stock, is the one with steady increment and instant gain. The value of the stock is in the discrete state space F, and the transition probability measure of the stock price process is set to b. The probability W is a positive random variable that can be viewed. Therefore, the option price at each moment can pass it in the same the conditional expected terminal value at probability n + 1 is given. The following will consider the situation where multiple beating factors occur simultaneously. First, Taylor expansion for A:

$$F_c = \frac{2b}{(n+l)K^{\frac{1}{n}}}\left(\frac{W}{A}\right)^{\frac{n+1}{n}} \tag{1}$$

The expected return rate b and volatility d of the stock are constructed as a function of time x. In addition, the jump variable R is modified in the model. In order to better control the jump intensity, a scaling factor is introduced to modify the jump term according to the formula:

$$\mathrm{d}F_r = \tau b dx \tag{2}$$

The pricing formula of exchange options under fractional jump-diffusion process can be obtained by establishing the model. In particular, under equal conditions, the pricing formula of exchange options under fractional Brownian motion environment can be obtained:

$$\mathrm{d}F_r = 2b\int_0^L \tau dx \tag{3}$$

Exchange option pricing in fractional Brownian motion

It is also assumed that it follows fractional Brownian motion. In the standard normal distribution cumulative distribution function, the jump process of stock price, the market is incomplete, and there is equivalent browse degree, but the browse degree is not only assumed that it is equal value, and found a sufficient and necessary condition. In this incomplete market, we can't completely wash off the risk. When the stock price falls for a long time due to abnormal business reasons, the company can take the method of "exchange option" to recover the old options that have been issued and replace them with new options. The arrival of information may cause a great change in the stock price, that is, if the stock price is in the first state at the time of state, then the stock price can change to any state except the first +1 and the first 1 at the time of + 1. We call this change a jump. On this basis, we modify the model to a process that can control the jump frequency, and further improve the expected return rate and volatility of the stock as continuous non-random functions of time and stock price. Revising the time interval of continuous investment portfolio and giving a clear definition can better replace the single term of investment. A continuous-time model with lognormal distribution as asset price distribution can produce the optimal portfolio principle.

Conclusion

In this paper, Taylor based jump diffusion model of fractional Brownian motion of stock price is studied. Economic development, especially financial innovation, makes the marketization of interest rate inevitable. Then there is the credit risk, which financial institutions must face. Fractional Brownian motion is not an ideal tool to describe the stock price process. Because of its self-similarity and long-term dependence, it has become a suitable tool for mathematical finance. If the jump form in the fractional jump diffusion process model is introduced into the generalized compensated Poisson process, the time ductility of the jump form can be well reflected. Under the condition of risk neutrality, based on fractional Brownian motion and a Poisson jump process, the probability of stock price changing to +l and 1 state increases when no significant information arrives, and at the same time the stock returns have the same distribution when significant information arrives. In practice, the jump of stock price may not necessarily follow the jump process, and the red interest rate may not necessarily be constant. Many more complicated situations need further study.

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